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RESEARCH REPORT

LOW VOLATILITY BETA ASYMMETRY: A CLOSER LOOK

ANALYSIS REVEALS HOW BETA ASYMMETRY WORKS – AND WHEN IT DOESN'T

The recent performance of low volatility strategies has challenged conventional wisdom by earning higher returns than benchmark portfolios at lower levels of volatility. This type of performance contradicts one of the most basic tenets of finance – that higher risk must accompany higher return. In our white paper "Low Volatility Investing: An Evolution in Alpha" we identify beta asymmetry as a key contributor to the recent success of low volatility strategies. Beta asymmetry refers to the tendency for the beta of low volatility portfolios to fall as market volatility rises (and vice-versa). It has become increasingly impactful given that the difference between down-market volatility and up-market volatility has grown considerably over time.[1] We attribute beta asymmetry to the volatility of the market portfolio increasing at a faster rate than the volatility of low volatility portfolios, and show the consistency of this relationship in the US, developed ex-US, and emerging markets.

In this paper we analyze beta asymmetry further by separating portfolio volatility into its two distinct components and evaluating each in turn. We find that changes to both the distribution of volatility (stock volatilities) and the level of diversification (stock correlations) are responsible for the beta asymmetry observed in low volatility portfolios. While both of these effects appear to scale linearly with market volatility, there is reason to believe beta asymmetry becomes nonlinear at some point. We conclude by assessing the key dependencies of beta asymmetry in periods of market stress, and note that the expected performance benefits fall as volatility becomes extreme.

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[1] The ratio of down-market volatility to up-market volatility for the Russell 1000 Index was 1.17 in the 1990s, 1.46 in the 2000s, and 1.77 in the 2010s, respectively.

A REVIEW OF BETA DYNAMICS

The beta of any portfolio may be represented as follows:

$$\beta_p = \rho_{p,m}(\sigma_p/\sigma_m)$$

Where:

- $\rho_{p,m}$ is the correlation between the portfolio returns R_p and the market returns R_m
- σ_p is the standard deviation of the portfolio returns R_p
- σ_m is the standard deviation of the market returns R_m

Portfolio beta is therefore a product of two factors, 1) the correlation of the portfolio to the market ($\rho_{p,m}$), and 2) the ratio of portfolio volatility to market volatility (σ_p/σ_m). Equity correlations tend to rise as market volatility increases, and the correlations of low volatility portfolios are no exception. However, in the case of low volatility, the ratio of portfolio volatility to market volatility (σ_p/σ_m) tends to *decline* as market volatility increases. In our original white paper we demonstrate this relationship empirically using daily data over the past 20 years. We repeat the analysis here, forming low volatility portfolios of stocks belonging to the bottom 30th percentile of daily return volatility, using look-back periods of three months, six months, and one year.[2] Portfolios are market capitalization-weighted and rebalanced quarterly (for methodology details refer to Appendix A).

The volatility ratio of low volatility portfolios *declines* with market volatility.

EXHIBIT 1: VOLATILITY RATIO SENSITIVITY TO MARKET VOLATILITY OF LOW VOLATILITY PORTFOLIOS IN THE RUSSELL 1000, MSCI WORLD EX US, AND MSCI EMERGING MARKETS INDEXES (12/31/1999 – 12/31/2019)

Slope coefficients and test statistics from regressions of the form:

$$(\sigma_{i_t}/\sigma_{m_t}) = \alpha_i + \beta_i(\sigma_{m_t}) + \varepsilon_{it}$$

Where:

- σ_i is the trailing 1-month standard deviation of the daily returns of low volatility portfolio i
- σ_m is the trailing 1-month standard deviation of the daily returns of market index portfolio m

	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility
Russell 1000	-7.86 [-6.15]	-8.01 [-6.11]	-7.84 [-5.86]
MSCI World ex US	-8.83 [-4.48]	-10.06 [-5.11]	-8.46 [-4.33]
MSCI Emerging Markets	-16.20 [-6.26]	-17.04 [-6.66]	-17.60 [-6.51]

Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

The negative, highly significant coefficients reported in Exhibit 1 provide evidence that the volatility ratio of low volatility portfolios declines with market volatility. In order to highlight the impact of this result, we compare average correlations, volatility ratios, and portfolio betas in "High" and "Low" volatility regimes[3] in Exhibit 2, and report the ratio of the two regime values as "Asymmetry" (High over Low).

[2] We depart from our original analysis by using daily return volatility and shorter look-back periods in order to more closely align with low volatility strategies in the marketplace.
[3] For volatility (risk) regime methodology details refer to Appendix A.

EXHIBIT 2: BETA OF LOW VOLATILITY PORTFOLIOS IN LOW AND HIGH VOLATILITY REGIMES (12/31/1999 – 12/31/2019)

Russell 1000 Low Volatility Portfolios

	Avg High Vol $\rho_{p,m}$	Avg Low Vol $\rho_{p,m}$	**$\rho_{p,m}$ Asymmetry**	Avg High Vol (σ_p/σ_m)	Avg Low Vol (σ_p/σ_m)	**(σ_p/σ_m) Asymmetry**	Avg High Vol β_p	Avg Low Vol β_p	**β_p Asymmetry**
3-Month Daily Volatility	0.90	0.90	**1.00**	0.76	0.87	**0.87**	0.69	0.78	**0.88**
6-Month Daily Volatility	0.90	0.91	**0.99**	0.76	0.87	**0.87**	0.69	0.79	**0.87**
1-Year Daily Volatility	0.90	0.91	**0.99**	0.77	0.88	**0.87**	0.70	0.80	**0.87**
Average	0.90	0.91	**0.99**	0.76	0.87	**0.87**	0.69	0.79	**0.87**

MSCI World ex US Low Volatility Portfolios

	Avg High Vol $\rho_{p,m}$	Avg Low Vol $\rho_{p,m}$	**$\rho_{p,m}$ Asymmetry**	Avg High Vol (σ_p/σ_m)	Avg Low Vol (σ_p/σ_m)	**(σ_p/σ_m) Asymmetry**	Avg High Vol β_p	Avg Low Vol β_p	**β_p Asymmetry**
3-Month Daily Volatility	0.93	0.93	**1.00**	0.73	0.84	**0.87**	0.68	0.78	**0.88**
6-Month Daily Volatility	0.93	0.92	**1.01**	0.73	0.85	**0.86**	0.68	0.78	**0.87**
1-Year Daily Volatility	0.93	0.92	**1.01**	0.73	0.84	**0.87**	0.69	0.78	**0.88**
Average	0.93	0.92	**1.01**	0.73	0.84	**0.87**	0.68	0.78	**0.88**

MSCI Emerging Markets Low Volatility Portfolios

	Avg High Vol $\rho_{p,m}$	Avg Low Vol $\rho_{p,m}$	**$\rho_{p,m}$ Asymmetry**	Avg High Vol (σ_p/σ_m)	Avg Low Vol (σ_p/σ_m)	**(σ_p/σ_m) Asymmetry**	Avg High Vol β_p	Avg Low Vol β_p	**β_p Asymmetry**
3-Month Daily Volatility	0.89	0.88	**1.00**	0.69	0.85	**0.81**	0.62	0.75	**0.82**
6-Month Daily Volatility	0.90	0.89	**1.02**	0.69	0.87	**0.80**	0.63	0.77	**0.81**
1-Year Daily Volatility	0.90	0.88	**1.01**	0.70	0.87	**0.80**	0.63	0.77	**0.82**
Average	0.89	0.88	**1.01**	0.69	0.86	**0.80**	0.62	0.76	**0.82**

Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

The results reveal that the change in the volatility ratio dominates the change in correlation, creating the beta asymmetry effect. In other words, the decline in the volatility ratio more than offsets the increase in correlation, causing beta to fall as market volatility rises. Given the prominent role it plays in beta asymmetry, the volatility ratio is the focus of analysis in this paper. In order to facilitate our analysis, we utilize a simple framework which separates portfolio volatility into two components: 1) the distribution of volatility and 2) the level of diversification.

PORTFOLIO VOLATILITY DECOMPOSED

There are two primary factors driving changes in volatility in the market portfolio. Changes to both 1) the volatility of individual stock returns, and 2) the correlations among stock returns, collectively determine the changes in market volatility on a period-over-period basis.[4] In order to separate the two effects we define diversification as the percentage decrease achieved in portfolio volatility relative to the portfolio-weighted average of the constituent stock volatilities:

$$\boldsymbol{D_p} = \left(\sum_{i=1}^{n} \boldsymbol{w_i \sigma_i} - \boldsymbol{\sigma_p}\right) / \sum_{i=1}^{n} \boldsymbol{w_i \sigma_i}$$

Where:

- D_p is the diversification of portfolio p
- w_i is the portfolio weight of stock i within portfolio p
- σ_i is the standard deviation of the returns of stock i within portfolio p
- σ_p is the standard deviation of the returns of portfolio p

[4] Constituent weights are also changing, though the net impact is typically de minimis to the volatility of the market portfolio.

Note that if every stock within a portfolio is perfectly correlated (i.e. equal to one) then the volatility of the portfolio is equal to the portfolio-weighted average of the individual stock volatilities and the diversification of the portfolio under this definition is zero. The value of diversification therefore represents the reduction in volatility attributable to non-perfect correlations of stocks held in a portfolio. Diversification is inversely related to correlations such that diversification increases as correlations decrease.

One should note that when utilizing this definition to compare diversification from one period to the next the net impact of changes to constituent correlations and weights are aggregated, requiring us to generalize conclusions to some extent. For large portfolios which are not heavily concentrated this type of analysis is generally suitable, though not infallible. For example, when diversification falls (rises) from one period to the next, we typically assume that correlations within the portfolio rose (fell) during the period. While this may be an accurate conclusion one cannot be certain without evaluating the underlying constituent details. Moreover to this point, if diversification remains the same from one period to the next it certainly does not imply that all pairwise stock correlations within the portfolio were unchanged. These considerations aside, this definition provides a simple way to quantify diversification, and gives us a way to cleanly separate the impact of changing volatilities from changing correlations.

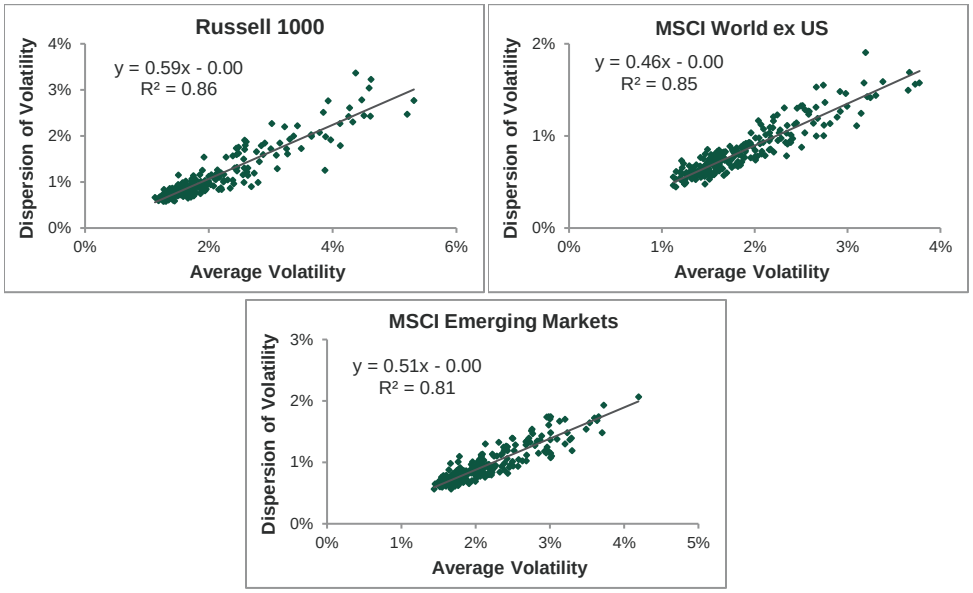
THE DISPERSION OF VOLATILITY

We begin our volatility ratio analysis by exploring how the distribution of volatility changes with market volatility. The volatility distribution characterizes the individual stock-level volatilities observed in the market over a given time period. The frequency of returns and look-back window length are defining characteristics of the distribution. In this paper, we compute the volatility distribution using total daily local returns and monthly look-back windows. The volatility distribution for any given month is therefore derived by first obtaining the total daily returns for all stocks belonging to the index during the month and then computing the standard deviation of returns for each individual stock. The volatility distribution characterizes the cross-sectional volatility of the stocks during the month, with the mean representing the average stock volatility and the standard deviation reflecting the dispersion of volatility in the market (i.e. the width of the distribution). Of course, volatility cannot be negative, and like many zero-bound variables the distribution of volatility is consistently positively skewed.[5]

A key feature of the volatility distribution is the strong relationship between the mean of the distribution and the dispersion of volatility. Exhibit 3 plots the paired monthly observations of the volatility distribution average and the dispersion of volatility for the past 2 decades in the Russell 1000, MSCI World ex US, and MSCI Emerging Markets Indexes. Linear trend lines, regression coefficients, and R^2 values are also included.[6]

[5] The average volatility distribution skew from 12/31/1999 to 12/31/2019 for the Russell 1000, MSCI World ex US, and MSCI Emerging Markets Indexes are 2.56, 2.36, and 1.70 respectively.
[6] In order to avoid the influence of outliers, observations with average monthly volatility in excess of 4 standard deviations have been removed. In each market, 2 of the 240 months have been removed for the purpose of estimating the models. All model output presented from this point forward applies the same 4 standard deviation threshold to independent variables for consistency.

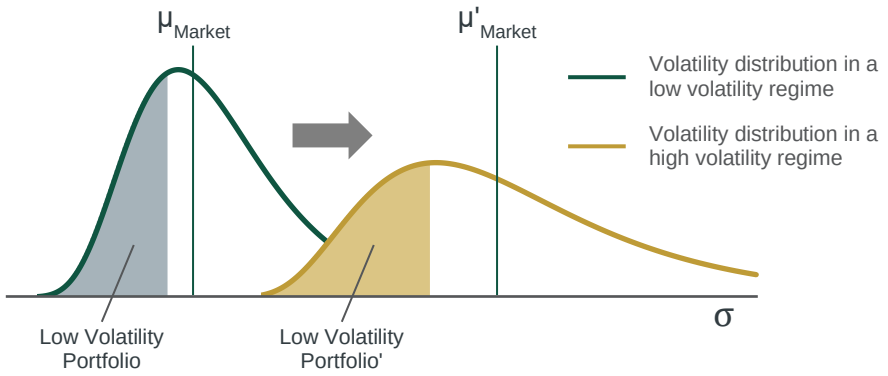
EXHIBIT 3: MEAN AND DISPERSION OF THE VOLATILITY DISTRIBUTION OF THE RUSSELL 1000, MSCI WORLD EX US, AND MSCI EMERGING MARKETS INDEXES (12/31/1999 – 12/31/2019)



SOURCE: Northern Trust Quantitative Research, FTSE Russell, MSCI

A cursory visual inspection of the graphs reveals a strong linear relationship between the average volatility and the dispersion of volatility in each market. The strength of this association is further corroborated by remarkably high R^2 values. Slope coefficients are similar across the markets, indicating the dispersion of volatility (cross-sectional standard deviation of stock volatilities) increases on average at a rate of 0.52% per every 1% increase in the average stock volatility. To summarize in the simplest of terms, the volatility distribution widens as it moves to the right. A stylized representation of this dynamic is depicted in Exhibit 4.

EXHIBIT 4: STYLIZED REPRESENTATION OF VOLATILITY DISPERSION INCREASING WITH VOLATILITY



The shaded region in each distribution represents the bottom 30% of stocks sorted by volatility and therefore corresponds to the methodology used to define low volatility portfolios in our analysis. As we can see from the graphic, the low volatility portfolio moves further from the mean as the distribution slides to the right – i.e. the space between the shaded regions and the lines representing the means (μ and μ') becomes greater. Consequently, one may be tempted to conclude that the difference between the average volatility of low volatility portfolios and the market portfolio gets larger as volatility increases. However, there is no guarantee that the constituents of the low volatility portfolio in a low volatility regime (left shaded region) are the same as the constituents in a high volatility regime (right shaded region). In fact, it is a near certainty that they will be different. Exhibit 5 reports the average monthly persistence of low volatility stocks, based on trailing 1-month daily volatility.[7] The table shows the percentage of low volatility stocks (bottom 30% of trailing 1-month volatility) that remain low volatility in the following month (bottom 30%, 40%, and 50% of forward 1-month volatility).

EXHIBIT 5: AVERAGE MONTHLY PERSISTENCE OF LOW VOLATILITY STOCKS (12/31/1999 – 12/31/2019)

	Trailing 1-Month Volatility	Forward 1-Month Volatility		
	Bottom 30%	Bottom 30%	Bottom 40%	Bottom 50%
Russell 1000	100%	63%	75%	83%
MSCI World ex US	100%	62%	73%	82%
MSCI Emerging Markets	100%	61%	72%	81%

Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

The data shows that over 60% of the low volatility stocks in any given month remain in the bottom 30% of volatility in the subsequent month, with over 80% remaining in the bottom half of volatility. This suggests that low volatility stocks tend to stay low volatility on a relative basis.[8] Given the degree of stability of low volatility portfolio constituents, it is indeed likely that the difference between the average volatility of the low volatility portfolio and the market portfolio increases with volatility, as Exhibit 4 leads us to conclude. However, this doesn't necessarily tell us anything about the *rate* of change.

With respect to the volatility ratio, the central question is whether the average volatility of the low volatility portfolio increases at a faster or slower rate than the average volatility of the market portfolio. As we will see, the main determining factor is the rate of increase in the dispersion of volatility. The average volatility of the low volatility portfolio increases as the volatility distribution moves to the right, but decreases as the distribution widens. The rate at which the distribution widens thereby largely determines how quickly the average volatility of the low volatility portfolio increases. We compare the average rates of change empirically using a similar model[9] as the one previously reported in Exhibit 1. The results for both equal-weighted and market capitalization-weighted (portfolio-weighted) averages are displayed in Exhibit 6 (for corresponding scatter plots

The widening of the volatility distribution slows the increase in volatility of low volatility portfolios.

[7] It is important to measure volatility stability using non-overlapping periods. For example, evaluating the monthly persistence of low volatility stocks defined with a 1-year look-back window is not insightful.
[8] Persistence of volatility is both expected and well-known (Engle, 1982).
[9] It is possible to analyze this analytically by imposing a distribution and modeling turnover as a function of volatility. We do not pursue this approach since the goal is to provide the reader with an intuition behind the empirical results.

refer to Appendix B). Equal-weighted averages are reflective of the volatility distribution and are consistent with the framework and analysis presented thus far (Exhibits 3 and 4). However, market cap-weighted averages reflect the constituent weighting scheme of the low volatility and market index portfolios[10], and are thus ultimately more relevant to the investor.

EXHIBIT 6: AVERAGE VOLATILITY RATIO SENSITIVITY TO AVERAGE MARKET VOLATILITY OF LOW VOLATILITY PORTFOLIOS IN THE RUSSELL 1000, MSCI WORLD EX US, AND MSCI EMERGING MARKETS INDEXES (12/31/1999 – 12/31/2019)

Slope coefficients and test statistics from regressions of the form:

$$(\mu_{i_t}/\mu_{m_t}) = \alpha_i + \beta_i(\mu_{m_t}) + \varepsilon_{it}$$

Where:

- μ_i is the trailing 1-month average daily stock volatility of low volatility portfolio i (equal-weighted and portfolio-weighted)
- μ_m is the trailing 1-month average daily stock volatility of market index portfolio m (equal-weighted and portfolio-weighted)

	Equal-weighted averages			Portfolio-weighted averages		
	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility
Russell 1000	-3.85 [-8.33]	-3.96 [-8.90]	-4.08 [-9.22]	-3.51 [-8.00]	-3.81 [-8.64]	-3.59 [-7.89]
MSCI World ex US	-2.22 [-4.10]	-1.90 [-4.01]	-1.90 [-4.32]	-3.27 [-6.11]	-3.23 [-6.34]	-2.60 [-5.36]
MSCI Emerging Markets	-2.95 [-4.17]	-3.14 [-5.37]	-3.36 [-6.05]	-3.08 [-4.67]	-3.17 [-5.04]	-2.63 [-3.73]

Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

Focusing first on the equal-weighted average results, we find that the coefficients are all negative and highly significant indicating that the average stock volatility of the low volatility portfolios increases at a slower rate than the average stock volatility of the market index portfolio. When comparing the average slope coefficient of each market, we observe that the Russell 1000 has the lowest value ([(-3.85) + (-3.96) + (-4.08)] / 3 = -3.96), followed by MSCI Emerging Markets (-3.15), and MSCI World ex US (-2.01), respectively. This rank ordering is consistent with the dispersion of volatility analysis reported previously. Recall that Exhibit 3 measured the rate of increase in the dispersion of volatility relative to the average volatility of the market. Stated differently, it quantified how quickly the distribution widened as it moved to the right (pushing the left tail of the distribution further from the mean). Referring back to the trend lines reported in Exhibit 3, we see that the slope coefficient is highest in the Russell 1000 Index (0.59), followed by the MSCI Emerging Markets Index (0.51), with the MSCI World ex US Index reporting the lowest value (0.46). Given that the distribution of volatility widened at the fastest rate in the Russell 1000 over the reported time period, it is logical that the average stock volatility of the low volatility portfolios in the Russell 1000 were the slowest to increase relative to the index.[11]

The portfolio-weighted average results support the conclusion that the volatility of low volatility portfolios increases at a slower rate than the volatility of the index, ignoring changes in diversification. While the impact of moving from an equal-

[10] The terms "market capitalization-weighted" and "portfolio-weighted" refer to the same methodology and are used interchangeably from this point forward.
[11] We do not interpret the relative ordering of the dispersion of volatility here, and therefore make no conclusions as to which market is expected to widen at the fastest rate moving forward.

weighted to market capitalization-weighted methodology changes the relative rank ordering of the average slope coefficient of each market[12], the magnitude and significance of the coefficients are collectively quite comparable. While acknowledging that the potential exists for equal-weighted and portfolio-weighted results to be materially different[13], the framework put forth is useful in understanding how the dispersion of volatility contributes to the beta asymmetry of low volatility portfolios.

Volatility distribution dynamics play a key role in beta asymmetry.

In this section we have shown that the portfolio-weighted volatility of low volatility portfolios increases at a slower rate than the portfolio-weighted volatility of the market. This distinguishing feature of low volatility portfolios is attributed to both the rate of increase in the dispersion of volatility and the stability of low volatility portfolio constituents. We began our volatility ratio analysis by separating portfolio volatility into two components so that each could be evaluated independently: 1) the distribution of volatility and 2) the level of diversification. Given the results presented thus far, we conclude that the beta asymmetry inherent in low volatility portfolios is due, at least in part, to volatility distribution dynamics. In the next section we analyze the level of diversification to determine the extent to which changing correlations contribute to beta asymmetry.

DIVERSIFICATION DECAY

The tendency for correlations to increase with volatility is one of the most widely accepted truths of finance. Its essence is captured by the common saying "in times of crisis, correlations go to one." As correlations increase, equity portfolios lose diversification. However, diversification loss does not affect all equity portfolios equally. Portfolios that exhibit high levels of diversification before an increase in volatility generally lose diversification more quickly than portfolios that are poorly diversified to begin with. At first pass this point may be counter-intuitive for many readers. However, it is important to separate the level of diversification from the speed of diversification loss (henceforth referred to as "diversification decay"). For example, it is possible (if not likely) for a well-diversified portfolio to suffer a much larger diversification loss than a concentrated portfolio during a volatility spike, yet still have a higher degree of diversification throughout the volatility event. In the context of analyzing the volatility ratio of low volatility portfolios this concept has direct implications. The higher the rate of diversification decay, the faster portfolio volatility increases. Should low volatility portfolios exhibit slower (faster) diversification decay than the market portfolio, then the decline in the volatility ratio will be accentuated (moderated).

Recall our formula for portfolio diversification, repeated here for convenience:

$$\boldsymbol{D_p} = \left(\sum_{i=1}^{n} \boldsymbol{w_i \sigma_i} - \boldsymbol{\sigma_p}\right) / \sum_{i=1}^{n} \boldsymbol{w_i \sigma_i}$$

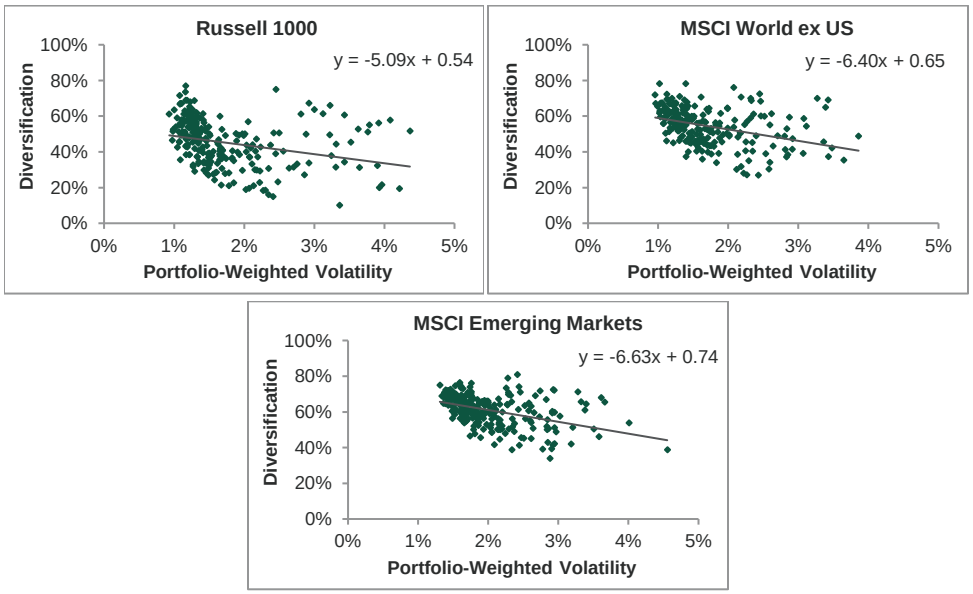
[12] The Russell 1000 has the lowest average market cap-weighted slope coefficient (-3.64), followed by MSCI World ex US (-3.03), and MSCI Emerging Markets (-2.96), respectively.
[13] For example, if the stocks with the largest market capitalization become concentrated in the bottom 30% of volatility, the portfolio-weighted average volatility of the market portfolio will converge towards those of the low volatility portfolios. Equal-weighted portfolios are unaffected by the distribution of market capitalization.

Where:

- D_p is the diversification of portfolio p
- w_i is the portfolio weight of stock i within portfolio p
- σ_i is the standard deviation of the returns of stock i within portfolio p
- σ_p is the standard deviation of the returns of portfolio p

Under this definition, the value of diversification represents the reduction in volatility attributable to non-perfect correlations of stocks held in a portfolio. Pairwise observations of index portfolio diversification (D_p) and index portfolio-weighted volatility[14] ($\sum_{i=1}^{n} w_i \sigma_i$) are plotted in Exhibit 7 for each distinct market, along with linear trend lines.

EXHIBIT 7: PORTFOLIO-WEIGHTED VOLATILITY AND DIVERSIFICATION OF THE RUSSELL 1000, MSCI WORLD EX US, AND MSCI EMERGING MARKETS INDEXES (12/31/1999 – 12/31/2019)



Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

The scatter plots confirm a negative relationship between diversification and market volatility, conforming roughly to a linear model.[15] In order to compare diversification decay rates, the diversification ratio of each low volatility portfolio to the market index portfolio (D_i/D_m) is regressed against the average market volatility (μ_m) and reported in Exhibit 8 (for corresponding scatter plots refer to Appendix C).

[14] In the special case of the market index portfolio, the portfolio-weighted average volatility ($\sum_{i=1}^{n} w_i \sigma_i$) is equal to the market capitalization-weighted average volatility (μ_m) of the market.

[15] Many readers may expect to see plots resembling an arc-cotangent function, where diversification flattens at the lowest and highest volatility levels with a downward sloping curve in-between. While the data shows virtually no support for this type of model, a power function of the form $y = ax^b$ results in a minor improvement in fit. We do not model the power function as the small improvement is immaterial to the analysis.

EXHIBIT 8: DIVERSIFICATION RATIO SENSITIVITY TO AVERAGE MARKET VOLATILITY OF LOW VOLATILITY PORTFOLIOS IN THE RUSSELL 1000, MSCI WORLD EX US, AND MSCI EMERGING MARKETS INDEXES (12/31/1999 – 12/31/2019)

Slope coefficients and test statistics from regressions of the form:

$$(D_{i_t}/D_{m_t}) = \alpha_i + \beta_i(\mu_{m_t}) + \varepsilon_{it}$$

Where:

- D_i is the diversification of low volatility portfolio i
- D_m is the diversification of market index portfolio m
- μ_m is the trailing 1-month average daily stock volatility of market index portfolio m (equal-weighted and portfolio-weighted)

	Equal-weighted averages			Portfolio-weighted averages		
	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility
Russell 1000	4.76 [5.53]	5.10 [5.69]	4.29 [4.72]	4.00 [3.90]	4.28 [4.00]	3.45 [3.20]
MSCI World ex US	5.88 [5.84]	6.91 [6.91]	5.93 [5.76]	5.19 [5.39]	6.17 [6.44]	5.19 [5.27]
MSCI Emerging Markets	6.18 [4.93]	6.74 [5.40]	6.44 [4.94]	5.63 [4.72]	6.14 [5.16]	6.05 [4.90]

Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

Slope coefficients for all portfolios are positive and highly significant, indicating that the diversification of low volatility portfolios falls more slowly as volatility increases than the diversification of the market index portfolio. In other words, the diversification decay rate of the market index portfolio is higher than each of the low volatility portfolios, thus accentuating the decline in the volatility ratio and enhancing beta asymmetry. To interpret this result, we refer back to the distribution of volatility. Recall that low volatility portfolios are concentrated on the left-side of the volatility distribution, and therefore avoid the most volatile stocks in the market. By contrast, the market portfolio includes the full volatility distribution. Since the diversification decay rate of the market portfolio is higher than the diversification decay rate of low volatility portfolios, one may conclude that the correlations of high volatility stocks are more sensitive to changes in market volatility (i.e. are less stable) than the correlations of low volatility stocks. Note that this doesn't necessarily indicate anything about which portfolio is more or less diversified on average (Appendix D reports the average diversification ratio (D_i/D_m) of the low volatility portfolios in Low, Mid, and High volatility regimes in each market).

These findings confirm that both components of portfolio volatility contribute to beta asymmetry: 1) the distribution of volatility (dispersion effect) and 2) the level of diversification (correlation effect). The fact that there are two aspects contributing independently to beta asymmetry attests to the robustness of the phenomenon. Of course, this does not guarantee that all volatility events result in favorable beta asymmetry outcomes for low volatility investors. In the next section we evaluate the key dependencies of beta asymmetry in order to identify potential points of failure.

Beta asymmetry is the result of both diversification decay and volatility distribution dynamics.

THE LIMITS OF BETA ASYMMETRY

While beta asymmetry has played a prominent role in the recent success of low volatility strategies, its outcome is far from certain. In order to understand the limits of beta asymmetry we evaluate some of the key drivers below, focusing specifically on extreme volatility events.

As volatility becomes elevated, any combination of the following may suppress beta asymmetry:

1. The persistence of low volatility stocks degrades. The dispersion of volatility effect relies on stability of low volatility constituents in order to contribute to the decline in the volatility ratio. According to the data in Exhibit 5, over 60% of low volatility stocks remain low volatility on a month-over-month basis, with over 80% remaining in the bottom half of volatility. During normal periods of volatility, idiosyncratic risk is an influential factor in determining low volatility constituents (i.e. stocks with the lowest amount of firm-specific risk are more likely to be found in low volatility portfolios). In times of stress, systematic risk begins to dominate, and low volatility constituents are determined largely by systematic risk exposures (e.g. industry and country exposures). This transition may result in significant turnover in low volatility constituents, diluting or even reversing the decline in volatility ratio that typically accompanies an increase in volatility.

2. Stock correlations reach an upper bound. The correlation effect is predicated on the level of diversification declining as volatility increases. The theoretical lower bound of diversification is zero, reached only when all pairwise stock correlations become one. In practice, the lower bound of diversification is a positive number. In the midst of a crisis, correlations in aggregate may reach a point in which they cannot practically move higher. In such a scenario, diversification becomes exhausted in the market (and by extension, within low volatility portfolios). By definition, the diversification decay rate becomes zero for all portfolios, forcing the diversification ratio to become constant. The correlation effect thus becomes neutralized, no longer contributing to beta asymmetry.

3. Low volatility portfolio correlations begin to differentiate. One of the most important enablers of beta asymmetry is the relatively static nature of low volatility portfolio correlations with the market. Exhibit 2 reports virtually no difference in portfolio correlations between “Low” and “High” volatility regimes, with all correlation asymmetry values very close to one. Upon review, this is not terribly surprising given that 1) the correlation between two large, well-diversified equity portfolios within the same market should always be high (e.g. 0.9), and 2) our volatility regime classifications are quite broad, with an equal number of months (80) belonging to each category. However, periods of stress may bring an exception to this norm. One can imagine a scenario in which the correlations of low volatility portfolios with the market begin to move materially higher as they converge towards one. Referring back to the formula for portfolio beta, we see that an increase in portfolio correlation moderates any decline in the volatility ratio.

Given the reasons cited above, it is plausible that beta asymmetry becomes neutralized entirely, or even *reverses*, beyond some level of volatility. This suggests that the beta of low volatility portfolios eventually bottoms, and possibly even increases as volatility continues to elevate. This does not necessarily imply that the expected beta of low volatility portfolios ultimately converges to one, thus eliminating all volatility reduction. However, it is reasonable to conclude that there are limits to beta asymmetry and its associated performance benefits.

The limits of beta asymmetry imply that extreme volatility may degrade low volatility performance.

CONCLUSION

The tendency for the beta of low volatility portfolios to fall as market volatility rises (and vice-versa) has been especially incremental to the performance of low volatility strategies over the past 20 years. During this time period, the difference between down-market volatility and up-market volatility has widened and the volatility-of-volatility has reached secular highs. Both of these market dynamics have been particularly constructive for low volatility strategies given their inherent beta asymmetry.

While beta asymmetry has been consistently observed across markets, it is not without limitations. In this paper, we identified two distinct components responsible for beta asymmetry – the distribution of volatility (dispersion effect) and the level of diversification (correlation effect). We then considered how each of these components may behave during times of stress in order to gain insight into the limitations of the phenomenon and the implications to low volatility strategy performance. We conclude that there is likely a volatility threshold beyond which beta asymmetry becomes neutralized and the associated performance benefits decline. This implies that the expected performance of low volatility strategies is conditioned on the level of volatility to some extent. It is important for investors to understand these dynamics in order to set expectations appropriately in the event of market turmoil.

APPENDIX A: METHODOLOGY DETAILS

Low volatility portfolio methodology

- All historical volatility variables are computed using equal-weighted[16] trailing total local returns
- Local returns are used in order to separate the low risk anomaly from currency effects
- Each risk variable is ranked within each region in order to limit the degree of bias on the analysis
- Region membership is determined in accordance with Fama and French (2012)
- Portfolios are rebalanced quarterly
    - All index constituents are sorted independently by each risk variable
    - The stocks in the bottom 30th percentile[17] of volatility are assigned to the corresponding low volatility portfolio and market capitalization-weighted

Volatility (risk) regime methodology

- Daily total local return volatility is computed for each market index portfolio every month from January 2000 through December 2019 (240 months)
- Months are sorted by index portfolio volatility to classify three volatility regimes (Low, Mid, and High) with an equal number of months (80) belonging to each classification

[16] A common alternative is to use an exponential-weighted methodology which gives greater emphasis to recent observations.
[17] The 30th percentile is commonly used to represent "high" or "low" portfolios in academic research. Similar results are obtained with quartile (25th percentile) or quintile (20th percentile) analysis.

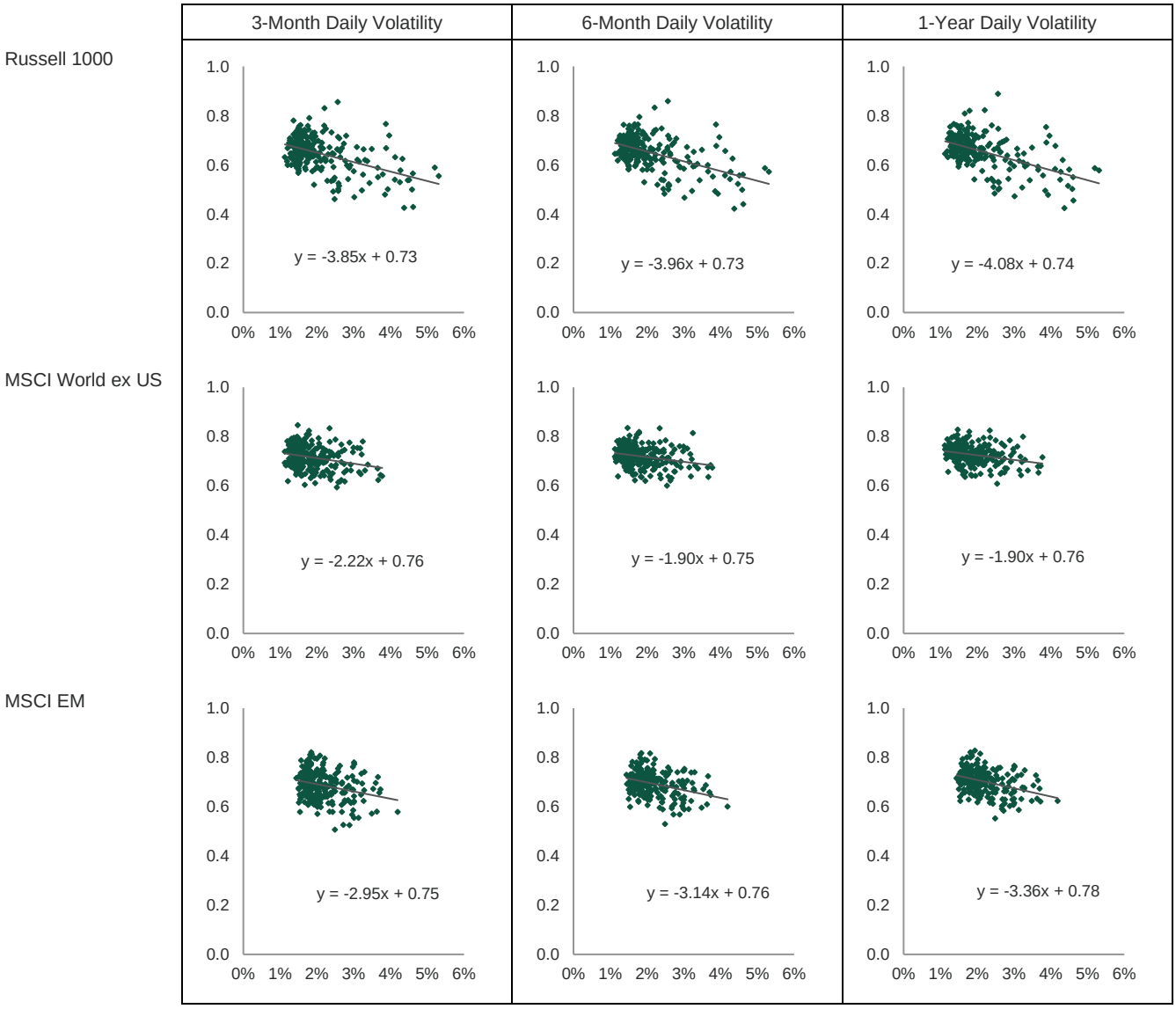
APPENDIX B: AVERAGE VOLATILITY RATIO SCATTER PLOTS OF LOW VOLATILITY PORTFOLIOS

EXHIBIT 9: EQUAL-WEIGHTED AVERAGE VOLATILITY RATIOS OF LOW VOLATILITY PORTFOLIOS IN THE RUSSELL 1000, MSCI WORLD EX US, AND MSCI EMERGING MARKETS INDEXES (12/31/1999 – 12/31/2019)

All scatter plots are of the form (x, y) = $(\mu_m,\ \mu_i/\mu_m)$

Where:

- μ_i is the trailing 1-month (equal-weighted) average daily stock volatility of low volatility portfolio i
- μ_m is the trailing 1-month (equal-weighted) average daily stock volatility of market index portfolio m



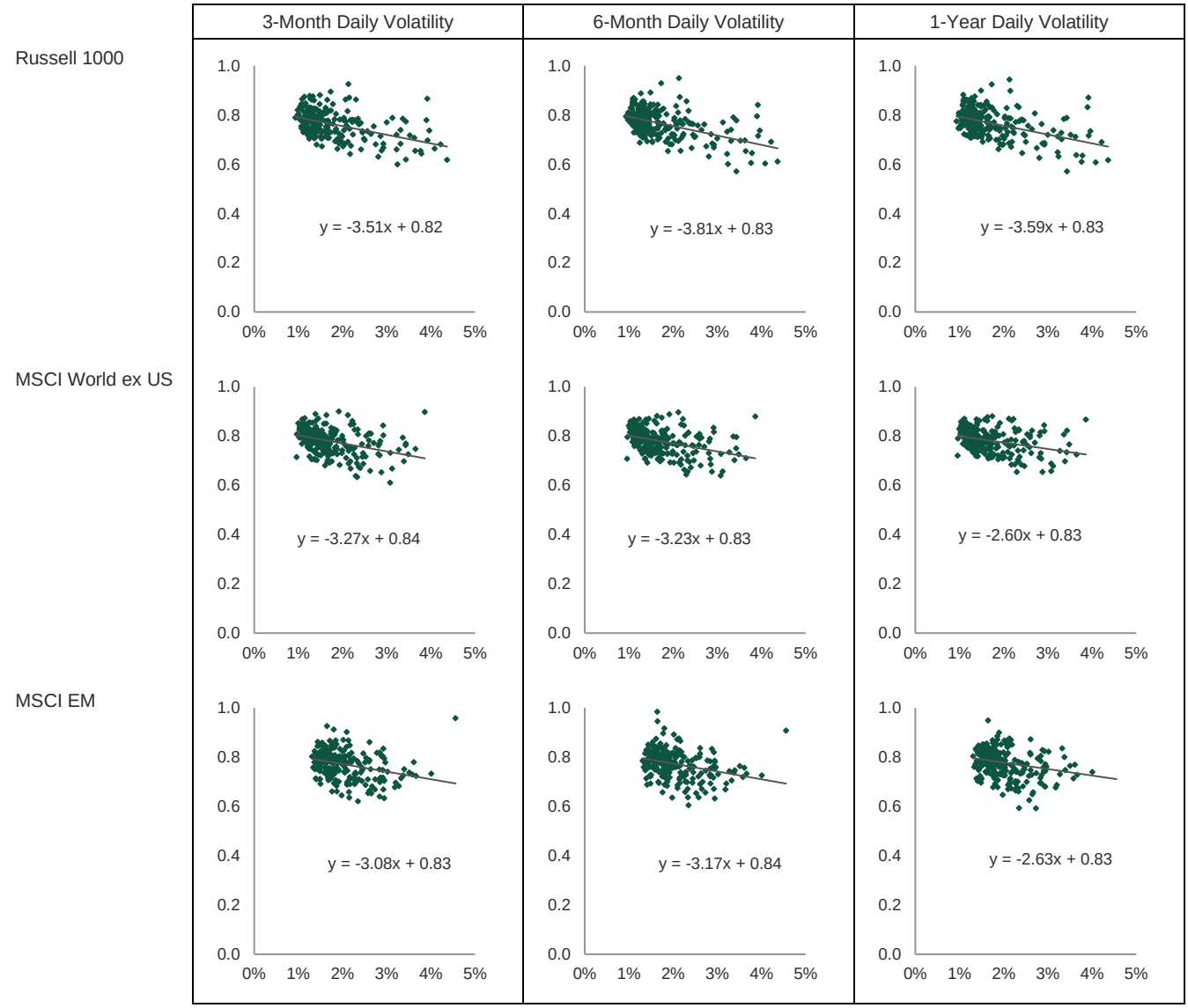
Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

EXHIBIT 10: PORTFOLIO-WEIGHTED AVERAGE VOLATILITY RATIOS OF LOW VOLATILITY PORTFOLIOS IN THE RUSSELL 1000, MSCI WORLD EX US, AND MSCI EMERGING MARKETS INDEXES (12/31/1999 – 12/31/2019)

All scatter plots are of the form (x, y) = $(\mu_m,\ \mu_i/\mu_m)$

Where:

- μ_i is the trailing 1-month (portfolio-weighted) average daily stock volatility of low volatility portfolio i
- μ_m is the trailing 1-month (portfolio-weighted) average daily stock volatility of market index portfolio m



Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

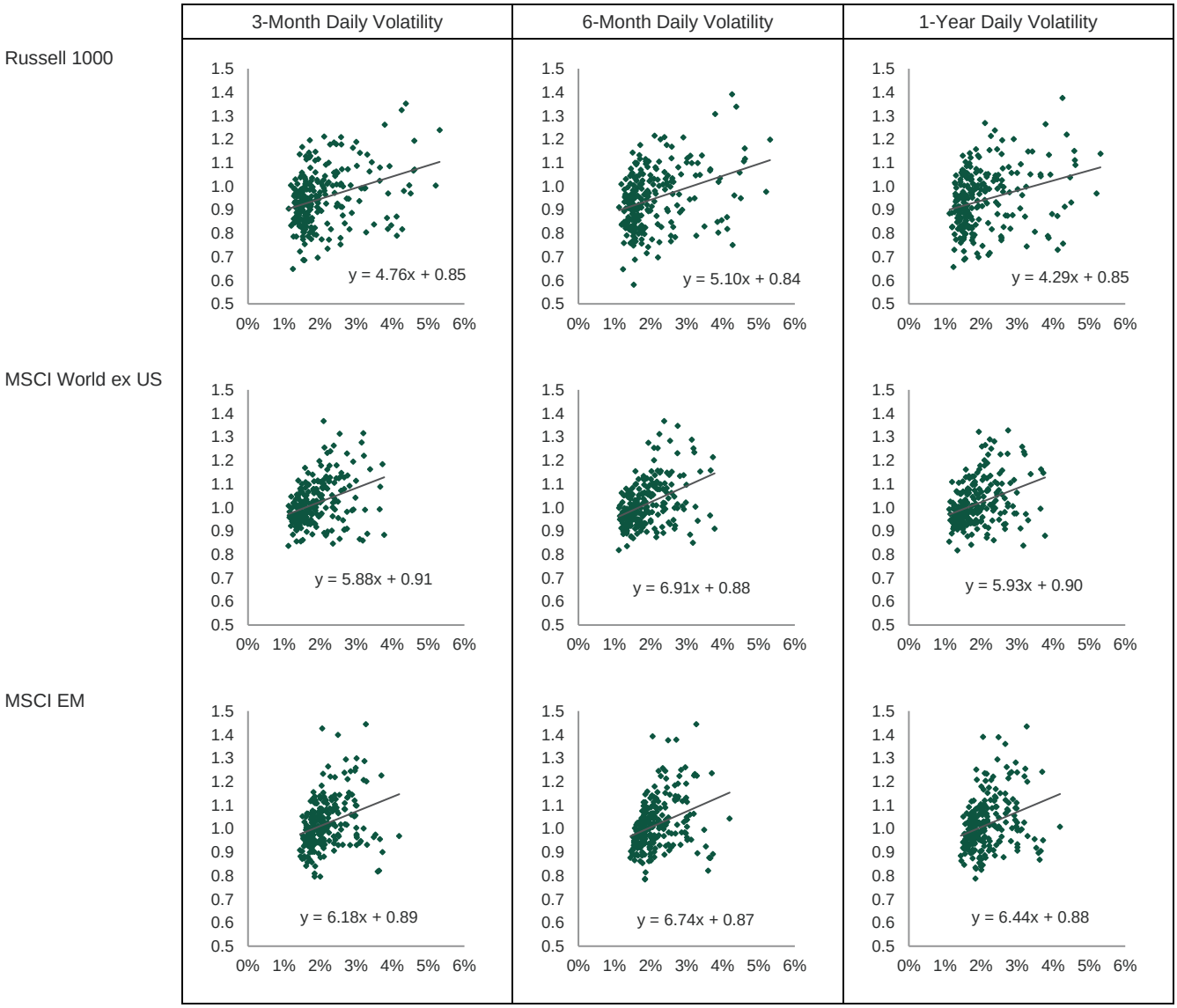
APPENDIX C: DIVERSIFICATION RATIO SCATTER PLOTS OF LOW VOLATILITY PORTFOLIOS

EXHIBIT 11: EQUAL-WEIGHTED DIVERSIFICATION RATIOS OF LOW VOLATILITY PORTFOLIOS IN THE RUSSELL 1000, MSCI WORLD EX US, AND MSCI EMERGING MARKETS INDEXES (12/31/1999 – 12/31/2019)

All scatter plots are of the form (x, y) = $(\mu_m,\ D_i/D_m)$

Where:

- D_i is the diversification of low volatility portfolio i
- D_m is the diversification of market index portfolio m
- μ_m is the trailing 1-month (equal-weighted) average daily stock volatility of market index portfolio m



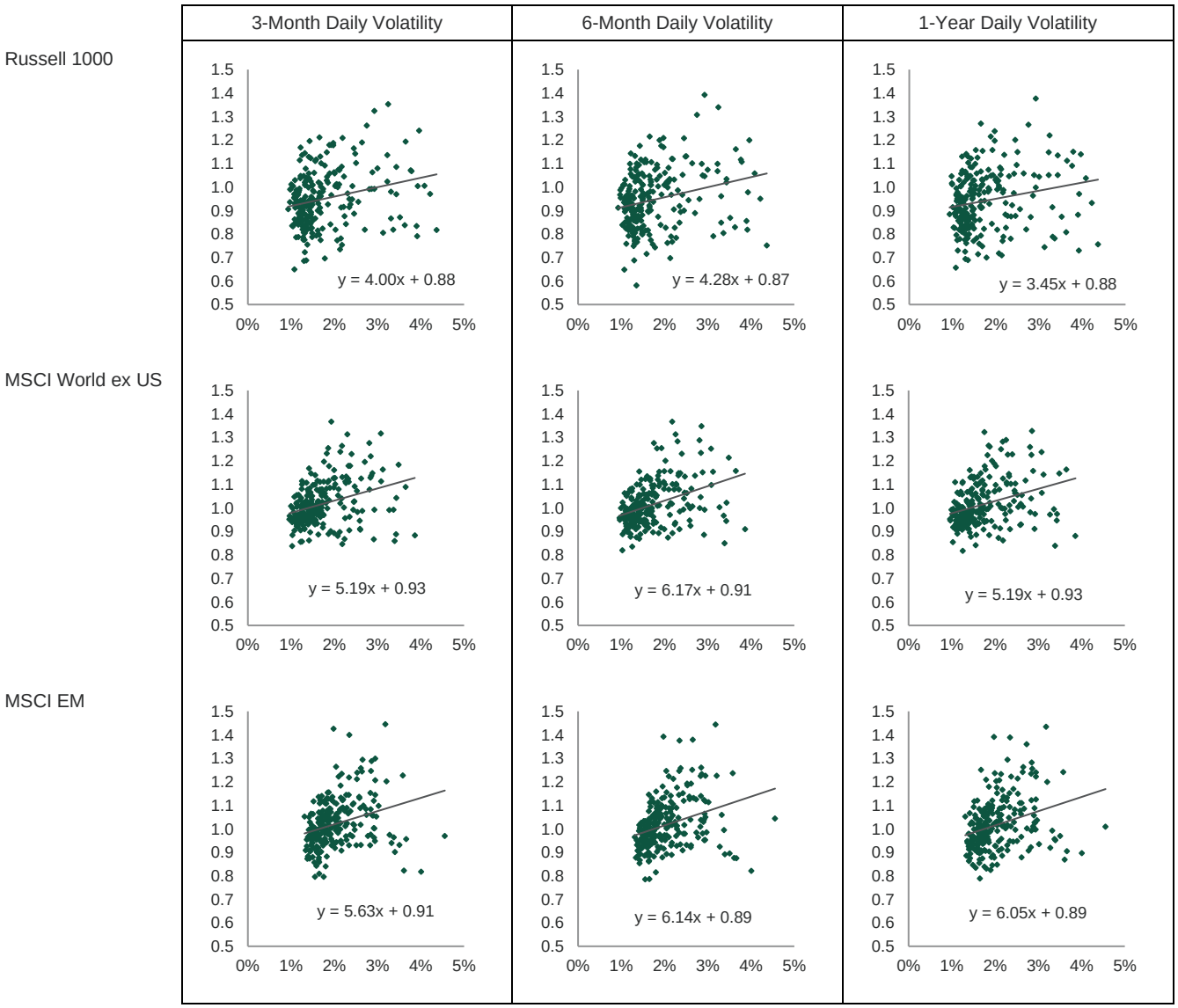
Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

EXHIBIT 12: PORTFOLIO-WEIGHTED DIVERSIFICATION RATIOS OF LOW VOLATILITY PORTFOLIOS IN THE RUSSELL 1000, MSCI WORLD EX US, AND MSCI EMERGING MARKETS INDEXES (12/31/1999 – 12/31/2019)

All scatter plots are of the form (x, y) = $(\mu_m,\ D_i/D_m)$

Where:

- D_i is the diversification of low volatility portfolio i
- D_m is the diversification of market index portfolio m
- μ_m is the trailing 1-month (portfolio-weighted) average daily stock volatility of market index portfolio m



Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

APPENDIX D: AVERAGE DIVERSIFICATION RATIOS OF LOW VOLATILITY PORTFOLIOS IN LOW, MID, AND HIGH VOLATILITY REGIMES

All months are sorted by the trailing 1-month average daily stock volatility of the market index portfolio (μ_m) under both equal-weighted and portfolio-weighted methods.[18] We then sort by μ_m to classify three volatility regimes (Low, Mid, and High) with an equal number of months (80) belonging to each classification. The average diversification ratio (D_i/D_m) is reported for each volatility regime.

Where:

- D_i is the diversification of low volatility portfolio i
- D_m is the diversification of market index portfolio m
- μ_m is the trailing 1-month average daily stock volatility of market index portfolio m (equal-weighted and portfolio-weighted)

EXHIBIT 13: AVERAGE DIVERSIFICATION RATIOS OF LOW VOLATILITY PORTFOLIOS IN LOW, MID, AND HIGH VOLATILITY REGIMES (12/31/1999 – 12/31/2019)

Russell 1000 Low Volatility Portfolios

	Equal-weighted averages			Portfolio-weighted averages		
	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility
Low	0.91	0.90	0.89	0.91	0.91	0.90
Mid	0.94	0.94	0.94	0.94	0.94	0.94
High	1.00	1.00	0.98	0.99	0.99	0.98

MSCI World ex US Low Volatility Portfolios

	Equal-weighted averages			Portfolio-weighted averages		
	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility
Low	0.97	0.97	0.97	0.97	0.97	0.97
Mid	1.01	1.01	1.01	1.01	1.00	1.01
High	1.06	1.06	1.05	1.06	1.06	1.06

MSCI Emerging Markets Low Volatility Portfolios

	Equal-weighted averages			Portfolio-weighted averages		
	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility	3-Month Daily Volatility	6-Month Daily Volatility	1-Year Daily Volatility
Low	0.97	0.96	0.97	0.96	0.96	0.97
Mid	1.02	1.02	1.01	1.03	1.03	1.01
High	1.07	1.07	1.07	1.07	1.07	1.07

Source: Northern Trust Quantitative Research, FTSE Russell, MSCI

In each market, the average diversification ratio for every low volatility portfolio is less than one (< 1) in the "Low" volatility regime. This indicates that the market index portfolio is more diversified than low volatility portfolios when market volatility is low. The relative influence of idiosyncratic risk is one possible explanation for this. When market volatility is low, the most volatile stocks may have the highest proportion of firm-specific risk which is effectively diversified away. The market portfolio is therefore more diversified than low volatility portfolios as it holds the entire volatility distribution. As market volatility increases, idiosyncratic risk begins to fall in favor of systematic risk and the difference in diversification between low volatility portfolios and the market portfolio declines. In the US, the average level of diversification of low volatility portfolios have approached that of the market portfolio but did not exceed it (i.e. all diversification ratios are <= 1). By contrast, the low volatility portfolios of the international markets have achieved

[18] Exhibit 2 classifies volatility regimes using index portfolio volatility (σ_m). In order to separate the level of diversification from the level of volatility, we use average daily stock volatility as a proxy for market volatility here.

higher diversification than the market portfolio in "High" volatility regimes (i.e. diversification ratios > 1). This result lacks obvious interpretation and is therefore somewhat surprising.

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IMPORTANT INFORMATION

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